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Low-temperature magnetic properties of a two-dimensional spin nematic state

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Abstract. A mean-field theory of the low-temperature magnetic properties of a 2D spin nematic (SN) state, which is one of the possible states of the weakly interacting electron system with a half-filled band on a square lattice, is considered. Such a state can result from a spin-triplet anisotropic electron–hole pairing and is characterized by circulating local spin currents violating translational symmetry of the underlying lattice in the absence of the charge-density wave or spin-density wave structures.

The existence of gapless quasiparticle excitations determines the peculiarities of the low-temperature behaviour of the 2D SN state. The 'relativistic' Landau quantization of the low-energy states in an external magnetic field results in anomalously strong diamagnetism, changing to paramagnetism on decreasing the angle between the field and the plane.

The orientational effect of the magnetic field and spin–orbit interaction on the spin vector $\boldsymbol{d}$ of the order parameter is studied. Different possibilities for the equilibrium orientation of the $\boldsymbol{d}$-vector are discussed.

1. Introduction

Recent studies of a weakly interacting electron system on a square lattice with a simple half-filled energy band, based on perturbative analysis (Dzyaloshinskii 1987, Dzyaloshinskii and Yakovenko 1988, Schulz 1987) have revealed the competition between superconducting, charge-density wave (CDW) and spin-density wave (SDW) instabilities, depending on the relation between the bare coupling constants. Later, it was shown (Nersesyan and Luther 1988, Schulz 1989a) that when, besides a point-like (Hubbard) interaction, finite-range and exchange interactions are also considered, the perfect nesting property of the square Fermi surface, corresponding to the case of an exactly half-filled band, can result in two more possible ordered states with divergent susceptibilities. Although translational symmetry of both states corresponds to doubling of the unit cell, it is not associated with the CDW or SDW structures. In fact, one of these order parameters, which describes an anisotropic spin-singlet electron–hole pairing and thus preserves spin rotational symmetry is characterized by non-zero local charge currents circulating around the plaquettes in such a way as to produce antiferromagnetically aligned local orbital moments. This is an orbital antiferromagnetic (OAF) state whose anomalous low-temperature properties have been recently discussed by Nersesyan and Vachnadze (1989). Another order parameter corresponds to a triplet anisotropic electron–hole pairing and describes a state in which spin-up and spin-down electron currents circulate around the plaquettes in opposite directions to produce non-zero spin currents. Having a zero local spin density, this state is only characterized by the global spin quantization axis, which breaks spin rotational symmetry and is analogous to the $\boldsymbol{d}$-vector of the order parameter in the superfluid ^{3}He-A. Such a state is called a spin nematic (SN) state (Andreev and Grischuk 1984) and will be investigated in this paper within a simple mean-field theory.

It should be pointed out that the possible existence of states with local currents has been previously noted by Halperin and Rice (1968) in a two-band model of an excitonic insulator. Recently the flux phase was studied (Affleck and Marston 1988, Kotliar 1988) in the 2D half-filled large-U Hubbard model. When the Mott limit of the exactly half-filled Hubbard model is considered (i.e. the case of the Heisenberg model), a local gauge symmetry appears as a result of the constraint to one particle at each lattice site. Because of this symmetry, the local currents in the flux phase are a gauge artefact and can only be observable at finite doping. On the other hand, in a weak-coupling theory, there is no constraint imposed on the particle number per site and hence no gauge symmetry. So, if under some conditions a SN (or OAF) phase occurs in a weakly interacting electron system with a half-filled band, the local spin (or charge) currents are non-zero owing to the symmetry properties of the corresponding order parameters and, of course, cannot be eliminated by a gauge transformation.

The notion of a SN state was introduced by Andreev and Grischuk (1984), who considered magnetic systems with exchange interactions, in which the magnetic ordering violates spin rotational symmetry but preserves time reversal invariance. The SN phase may be realized, for example, in quantum spin-1 systems with competing quadratic and biquadratic exchange interactions (Andreev and Grischuk 1984, Papanicolaou 1988). A parity-violating twisted SN state has recently been discussed in a profound paper by Chandra *et al* (1990), where a quantum fluid approach to frustrated 2D exchange magnets is developed. Since in such systems the average local spin density vanishes, the order parameter can only be introduced as a characterization of transformation properties of two-spin correlation functions. It was pointed out (Andreev and Grischuk 1984) that exchange SN phases do not differ qualitatively from antiferromagnets with respect to macroscopic magnetic properties.

On the other hand, our weak-coupling 2D model, discussed below, represents a fermionic realization of the SN state, in which the order parameter is introduced as the amplitude of anomalous electron–hole pairing with a special spin and space structure. The peculiar feature of the fermionic SN state is the existence of gapless single-fermion excitations associated with zeros of the gap function on the Fermi surface. The appearance of these zeros is a direct consequence of violation of the point symmetry of the underlying lattice in the SN state. At low temperatures, the low-energy fermion excitations contribute to power-law temperature dependence of thermodynamic quantities. They play a minor role, when a magnetic field is parallel to the plane of the system, in which case the magnetic properties of the SN state are qualitatively the same as those of the SDW state.

As we shall show, the main distinctive property of the fermionic SN state is revealed at finite inclination of the field, when the peculiar Landau quantization of the 'relativistic' fermion spectrum takes place. As in the 2D OAF (Nersesyan and Vachnadze 1989), in this case strong diamagnetism, with unusual temperature and magnetic field dependence of the magnetic susceptibility, is observed. The 'relativistic' Landau quantization also leads to an interesting orientational effect of the magnetic field and spin–orbit interaction on the $\boldsymbol{d}$-vector. We show that the Zeeman splitting of the zeroth Landau level gives an important contribution to the anisotropy energy, linear in the components of the $\boldsymbol{d}$-vector. Together with a contribution from higher-energy states, this results in different possibilities for the equilibrium orientation of the $\boldsymbol{d}$-vector, which can be continuously changed on varying the angle between the magnetic field and the plane of the system. We also discuss the role of the spin–orbit interaction and show that its combined effect with 'relativistic' Landau quantization is revealed in a contribution to the anisotropy energy, linear in both the magnetic field and the spin–orbital coupling constant, resulting in a two-axis in-plane anisotropy for the $\boldsymbol{d}$-vector.

2. Order parameter and mean-field Hamiltonian

We consider the electron system on a square lattice with a simple, orbitally non-degenerate tight-binding spectrum $\varepsilon(\boldsymbol{k}) = -2t[\cos(k_x a) + \cos(k_y a)]$, formed by the electron hopping between the nearest-neighbour sites. Throughout this paper the energy band is assumed to be exactly half filled, in which case the Fermi surface is a perfect square and has the nesting property: $\varepsilon(\boldsymbol{k} + \boldsymbol{Q}) = -\varepsilon(\boldsymbol{k})$, where $\boldsymbol{Q} = (\pi/a, \pi/a)$. When the interaction between electrons is taken into account, this property can result in anomalous electron–hole pairing with the order parameter

$$A_{\alpha\beta}(\boldsymbol{k}) = \langle c^{+}_{\boldsymbol{k}\alpha} c_{\boldsymbol{k}+\boldsymbol{Q},\beta} \rangle \tag{1}$$

where α and β are spin variables.

As pointed out by Nersesyan and Luther (1988), in the SN state, the point symmetry D_4 of the square lattice is broken down into the subgroup D_2, including rotations through an angle π about the horizontal x and y axes and vertical z axis. Spin rotational symmetry SU(2) is broken down to U(1), while time reversal invariance is preserved. In the simplest case of the electron–hole pairing on the neighbouring sites of the lattice, the SN order parameter, satisfying the above symmetry conditions, has the form

$$A_{\alpha\beta}(\boldsymbol{k}) = \mathrm{i}(\boldsymbol{d} \cdot \boldsymbol{\sigma}_{\alpha\beta}) A_0(T)[\cos(k_x a) - \cos(k_y a)] \tag{2}$$

where $A_0(T)$ is a real temperature-dependent amplitude, $\boldsymbol{\sigma}$ are the Pauli matrices and $\boldsymbol{d}$ is a unit vector in the spin space. Factorization of the order parameter (2) suggests the limit of weak spin–orbital coupling. In the absence of an external magnetic field and spin–orbit interaction, the space of degeneracy for the $\boldsymbol{d}$-vector is a two-dimensional sphere S^2 of unit radius†.

The symmetry properties of the order parameter (2) imply that there is no density or spin-density modulation as well as no charge currents in the SN state. The physical characterization of this state is the existence of non-zero spin currents on the links of the lattice. Choosing $\boldsymbol{d} = \hat{z}$, for spin currents flowing through the links $\langle \boldsymbol{n}, \boldsymbol{n} \pm \boldsymbol{a}_x \rangle$ and $\langle \boldsymbol{n}, \boldsymbol{n} \pm \boldsymbol{a}_y \rangle$ ($\boldsymbol{a}_x$ and $\boldsymbol{a}_y$ being the basis vectors of the square lattice), one finds that

$$\begin{aligned} j^{\text{spin}}_{\boldsymbol{n},\boldsymbol{n}\pm \boldsymbol{a}_x} &\sim \pm\, t A_0 (-1)^{n_x + n_y} \boldsymbol{a}_x \\ j^{\text{spin}}_{\boldsymbol{n},\boldsymbol{n}\pm \boldsymbol{a}_y} &\sim \mp\, t A_0 (-1)^{n_x + n_y} \boldsymbol{a}_y . \end{aligned} \tag{3}$$

† Note that, if $\boldsymbol{d}$ is changed by $-\boldsymbol{d}$, the order parameter (2) changes its sign. This change can be compensated by a proper transformation from the space group of the original (disordered) system (e.g. by a $\pi/2$ rotation $k_x \to k_y$, $k_y \to -k_x$), or by changing the sign of the amplitude A_0. However, since all these compensating transformations belong to a discrete group, there is no continuous path on S^2 connecting the points $\boldsymbol{d}$ and $-\boldsymbol{d}$, which therefore cannot be identified. Therefore, in contrast with the superfluid ^{3}He–A (see e.g., Volovik 1984) there are no half-vortices among topologically stable configurations of the vector field $\boldsymbol{d}(\boldsymbol{x})$.

Equations (3) clearly show that spin-up and spin-down currents circulate around each plaquette in opposite directions and their distribution over the lattice corresponds to doubling of the unit cell.

The simplest mean-field microscopic Hamiltonian which describes a homogeneous ordered SN state has the form

$$\hat{H} = \sum_{k} [\varepsilon(k) c_{k\alpha}^{+} c_{k\alpha} - \mathrm{i}\Delta(k)(d \cdot \sigma_{\alpha\beta}) c_{k\beta}^{+} c_{k+Q,\alpha}] \tag{4}$$

where the gap function

$$\Delta(k) = \Delta_0(T)[\cos(k_x a) - \cos(k_y a)]. \tag{5}$$

Here Δ_0 ($\Delta_0 \ll W$, where $W = 8t$ is the bandwidth of the original tight-binding spectrum) is a real amplitude which is assumed to be non-zero below some characteristic mean-field temperature. In what follows, only the low-temperature region $T \ll \Delta_0$ will be considered.

We shall not analyse the self-consistency equation for Δ_0, which requires the detailed knowledge of interactions in the system, as well as taking into account the interference between different competing instabilities (Dzyaloshinskii 1987, Dzyaloshinskii and Yakovenko 1988, Schulz 1987). Nonetheless, it should be pointed out that the development of such anisotropic correlations like the SN and OAF ones is only possible if the effective interaction is not point-like (Nersesyan and Luther 1988). This requirement can also be understood from the dynamical point of view. In fact, the appearance of local currents in the weak-coupling SN (or OAF) phase at exactly half-filling implies that doubly occupied configurations enter the picture. On the other hand, the Hubbard on-site (although weak) repulsion leads to the tendency towards a SDW ordering. To overcome this tendency and to stabilize the currents, one has to take into account finite-range interactions.

This can be demonstrated within a half-filled extended 'U–V–J' Hubbard model including direct (V) and exchange (J) interactions on the neighbouring sites (U, $V > 0$). Making a RPA estimation of the mean-field transition temperatures for different instabilities, we show in the appendix that, in the vicinity of the critical line $U = 4V$, which separates the SDW ($U > 4V$) and CDW ($U < 4V$) phases (Zhang and Callaway 1989), the SN instability turns out to be dominant if the exchange constant $J < 0$ (at $J > 0$ the dominant instability is the OAF one). Under these conditions the 'U–V–J' model provides a justification for the mean-field Hamiltonian (4).

The spectrum of the Hamiltonian (4) is identical with the spectrum of the mean-field OAF Hamiltonian (Nersesyan and Vachnadze 1989):

$$E_{\pm}(k) = \pm E_0(k) \qquad E_0(k) = [\varepsilon^2(k) + \Delta^2(k)]^{1/2}. \tag{6}$$

Since the original tight-binding band is half filled, the exact electron–hole symmetry implies that, in the ground state of the system, the lower band is filled, while the upper band is empty. The gap between the bands vanishes at two inequivalent points, $k_1 = (\pi/2a, \pi/2a)$ and $k_2 = (-\pi/2a, \pi/2a)$; so the SN state, as well as the OAF state, represents a 2D Fermi system with a zero-dimensional Fermi surface. In the low-energy region ($|E| \ll |\Delta_0|$), the spectrum contains two branches of gapless quasiparticle excitations with a 'relativistic' (i.e. linear) dispersion relation. The Hamiltonian of these excitations, which represents the continuum limit of the original lattice model (4), has the form

$$
\begin{aligned}
H &= H_1 + H_2 \\
H_1 &= \int \mathrm{d}^2x\, \psi^+_{1\alpha}(x)[c_\parallel \delta_{\alpha\beta}\hat{p}_x\tau_x + c_\perp(\boldsymbol{d}\cdot\boldsymbol{\sigma}_{\alpha\beta})\hat{p}_y\tau_y]\psi_{1\beta}(x) \\
H_2 &= \int \mathrm{d}^2x\, \psi^+_{2\alpha}(x)[c_\parallel \delta_{\alpha\beta}\hat{p}_y\tau_x + c_\perp(\boldsymbol{d}\cdot\boldsymbol{\sigma}_{\alpha\beta})\hat{p}_x\tau_y]\psi_{2\beta}(x)
\end{aligned}
\tag{7}
$$

where, for a given spin projection, $\psi_{1\alpha}$ and $\psi_{2\alpha}$ are two-component spinor fields describing electron states near $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, respectively, τ_i are the Pauli matrices in the spinor space, and group velocities

$$
c_\parallel = 2\sqrt{2}ta \qquad c_\perp = \sqrt{2}\Delta_0 a. \tag{8}
$$

In obtaining equations (7) a $\pi/4$ rotation of the coordinate system in real space was done.

Since the quasiparticle density of states is a linear function of the energy, i.e. $\rho(\varepsilon) \sim |\varepsilon|$, the electron compressibility and specific heat show power-law asymptotic behaviour as $T \to 0$, $\kappa \sim T/W\Delta_0$, $C \sim T^2/W\Delta_0$, which are essentially the same as in the OAF state. These power laws distinguish the SN and OAF states from the CDW and SDW states; in the latter states the absence of zeros in the corresponding gap functions leads to an exponentially fast decrease in thermodynamic quantities with the temperature. The most interesting low-temperature magnetic properties of the SN state will be considered in the rest of this paper.

3. Magnetic properties in a parallel magnetic field

In this section we consider the case of a magnetic field parallel to the plane of the system to calculate the spin susceptibility tensor and thus to determine the equilibrium orientation of the $\boldsymbol{d}$-vector in such a field. The parallel magnetic field has no influence on the orbital motion of the particles; so only the term describing the Zeeman splitting of the single-particle states, namely

$$
H_Z = \sum_{\boldsymbol{k}} \boldsymbol{h}\cdot\boldsymbol{\sigma}_{\alpha\beta} c^+_{k\alpha} c_{k\beta} \tag{9}
$$

should be added to the Hamiltonian (4). Here $\boldsymbol{h} = \mu_B \boldsymbol{H}$ ($|\boldsymbol{h}| \ll \Delta_0$), μ_B being the Bohr magneton.

At $\boldsymbol{h} \neq \boldsymbol{0}$, the spectrum is modified as follows:

$$
E^2(\boldsymbol{k}) = h^2 + \varepsilon^2(\boldsymbol{k}) + \Delta^2(\boldsymbol{k}) \pm 2[h^2\varepsilon^2(\boldsymbol{k}) + (\boldsymbol{d}\cdot\boldsymbol{h})^2\Delta^2(\boldsymbol{k})]^{1/2}. \tag{10}
$$

When $\boldsymbol{d}$ and $\boldsymbol{h}$ are parallel, the Zeeman splitting of the zero-field spectrum (6) takes place:

$$
E_{\pm,\sigma}(\boldsymbol{k}, \boldsymbol{\sigma}) = E_\pm(\boldsymbol{k}) + \sigma h \qquad (\sigma = \pm 1) \tag{11}
$$

resulting in the appearance of a finite Fermi surface and a finite density of states at the Fermi level, proportional to the magnetic field. Therefore, at $T = 0$, the longitudinal spin susceptibility $\chi_\parallel \sim |\boldsymbol{h}|$, tending to zero with the field. At finite temperatures ($T \ll \Delta_0$), the zero-field susceptibility is a linear function of the temperature, i.e. $\chi_\parallel \sim T$, as opposed to the SDW state, where $\chi_\parallel$ is exponentially small.

When $\boldsymbol{d}$ is perpendicular to $\boldsymbol{h}$, equation (10) yields

$$E_{\pm,\sigma}(\boldsymbol{k}; h) = \pm \{[\varepsilon(\boldsymbol{k}) + \sigma h]^2 + \Delta^2(\boldsymbol{k})\}^{1/2}. \tag{12}$$

Here the magnetic field plays the role of a chemical potential which determines two new Fermi surfaces of the original (disordered) system, $\varepsilon(\boldsymbol{k}) = \pm h$. In this case the Fermi surface in the ordered SN state still remains zero dimensional, but the number of zeros now equals four, since the magnetic field splits each of the two zeros of the original spectrum (6). Therefore, in the low-energy region ($|E| \ll \Delta_0$), where the picture of gapless 'relativistic' excitations is valid, the magnetic field can be eliminated by a gauge transformation. So the transverse susceptibility $\chi_\perp$ will mainly be contributed by the region of higher energies ($\Delta_0 \ll |E| \ll W$), and hence will be comparable with the paramagnetic susceptibility of the normal metallic state with a half-filled band. A simple calculation shows that, at $T \ll \Delta_0$, within the logarithmic accuracy, the zero-field transverse susceptibility equals

$$\chi_\perp = \sum_{\boldsymbol{k}} \left[\frac{2\Delta^2(\boldsymbol{k})}{E_0^3(\boldsymbol{k})} \tanh\left(\frac{E_0(\boldsymbol{k})}{2T}\right) + \frac{1}{T}\frac{\varepsilon^2(\boldsymbol{k})}{E_0(\boldsymbol{k})}\frac{1}{\cosh^2[E_0(\boldsymbol{k})/2T]}\right]$$
$$= \frac{8N}{\pi^2 W}\left[\ln\left(\frac{W}{\Delta_0}\right) + \mathcal{O}\left(\frac{T}{\Delta_0}\right)\right] \tag{13}$$

where the logarithmic factor is due to the Van Hove singularity of the density of states at the Fermi surface of the metallic phase. The change in the free energy in the magnetic field is

$$\Delta F(H) = F(H) - F(0) = -\tfrac{1}{2}\chi_\perp h^2 - \tfrac{1}{2}(\chi_\parallel - \chi_\perp)(\boldsymbol{d}\cdot\boldsymbol{h})^2. \tag{14}$$

From (14) it follows that, since $\chi_\perp \gg \chi_\parallel$, in the magnetic field parallel to the plane of the system, the $\boldsymbol{d}$-vector must be oriented in the plane perpendicular to the field. There is a degeneracy with respect to two-dimensional rotations of the $\boldsymbol{d}$-vector in this plane; so the space of degeneracy is reduced to a circle S^1.

4. Magnetic properties in an arbitrarily oriented magnetic field and anisotropy energy

In this section we shall consider the case when the magnetic field makes a finite angle $\alpha (0 < \alpha < \pi/2)$ with the plane of the system, affecting the orbital motion of the particles, and calculate the magnetic field dependence of the ground-state energy. Now, to describe the low-temperature properties of the SN state, one must know the exactly quantized spectrum of low-energy excitations, since at $T = 0$ the wavelength of these excitations diverges on approaching the Fermi points $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$. As in the OAF phase (Nersesyan and Vachnadze 1989), a 'relativistic' quantization of the Landau levels in the presence of a finite magnetic flux through the plane (Jackiw 1984, Balatsky *et al* 1986)

$$E_n^\pm = \pm\sqrt{|\Omega_\perp| n} \qquad (n = 0, 1, 2, \ldots) \tag{15}$$

where

$$\Omega_\perp = 2eH_\perp c_\parallel c_\perp / c \tag{16}$$

($H_\perp = H \sin\alpha$ being a component of the field, perpendicular to the plane) results in anomalously strong diamagnetism, with unusual magnetic field and temperature dependence of the diamagnetic susceptibility.

Certainly, the applicability of (15) is restricted by the condition $|\Omega_\perp|^{1/2} \ll \Delta_0$, which, due to its order of magnitude, reduces to $(h/\Delta_0)^{1/2} \ll 1$. Therefore, if the angle α is not too small, the following condition is satisfied

$$h/|\Omega_\perp| \sim (h/\Delta_0)^{1/2} (\sin\alpha)^{-1/2} \ll 1 \tag{17}$$

implying that the spacing between the $n=0$ and $n=1$ Landau levels considerably exceeds the Zeeman splitting. Then only splitting of the $n=0$ Landau level, located at the Fermi energy, should be taken into account. As we shall see, this results in an important contribution to the anisotropy energy at $T=0$, linear in components of the $\boldsymbol{d}$-vector.

Let us first consider the contribution of the low-energy region $|E| \ll \Delta_0$ to the ground state energy. In the absence of spin–orbit interaction, real and spin spaces are completely decoupled. It is then convenient to direct the $\boldsymbol{d}$-vector along the spin z-axis and to choose $\boldsymbol{h}$ in the xz-plane of the spin space, $\boldsymbol{h} = (h_x, 0, h_z)$. Making a $\pi/4$ rotation of the real space, choosing a Landau gauge for the vector-potential, $\boldsymbol{A} = (0, H_\perp x)$, and transforming

$$\psi_2(x) \to \psi_2(x)\exp(ieH_\perp xy/c)$$

to obtain the effective gauge $\boldsymbol{A} = (-H_\perp y, 0)$ for the second group of fermions, we get

$$H_1 = \int d^2x\, \psi_1^+(x)\hat{\mathcal{H}}_1(x)\psi_1(x) \qquad H_2 = \int d^2x\, \psi_2^+(x)\hat{\mathcal{H}}_2(y)\psi_2(x) \tag{18}$$

where the one-dimensional, formally relativistic Hamiltonians $\hat{\mathcal{H}}_1$ and $\hat{\mathcal{H}}_2$ have the form

$$\hat{\mathcal{H}}_1(x) = c_\parallel \hat{p}_x \tau_x - (eH_\perp c_\perp \tilde{x}/c)\sigma_z\tau_y - h_x\sigma_x - h_z\sigma_z \tag{19}$$

$$\hat{\mathcal{H}}_2(y) = c_\parallel \hat{p}_x \tau_x + (eH_\perp c_\perp \tilde{y}/c)\sigma_z\tau_y - h_x\sigma_x - h_z\sigma_z \tag{20}$$

where $\tilde{x} = x - ck_y/eH_\perp$, $\tilde{y} = y + ck_x/eH_\perp$, k_y and k_x being the conserved components of the momentum for the two gauges, respectively.

Note that the Hamiltonian remains invariant under the transformations

$$x \to y \qquad y \to -x \qquad \psi_1 \to \tau_x\psi_2 \qquad \psi_2 \to \tau_y\psi_1 \qquad \mathcal{H}_1 \leftrightarrow \mathcal{H}_2 \tag{21}$$

and so it is sufficient to consider only the Hamiltonian $\mathcal{H}_1$.

Using a gauge transformation $\psi_1(x,y) \to U^{-1}(x)\psi_1(x,y)$ with

$$U(x) = \exp[-i(h_x x/c_\parallel)\sigma_x\tau_x]$$

yields

$$\mathcal{H}_1(x) \to U(x)\mathcal{H}_1(x)U^{-1}(x) = \mathcal{H}_1^{(0)}(x) + \mathcal{H}_1^{(1)}(x) \tag{22}$$

where

$$\mathcal{H}_1^{(0)} = c_\parallel \hat{p}_x\tau_x - (eH_\perp c_\perp \tilde{x}/c)\sigma_z\tau_y - h_z\sigma_z \tag{23}$$

$$\mathcal{H}_1^{(1)} = h_z\{\sigma_z[1 - \cos(2h_x x/c)] + \sigma_y\tau_x \sin(2h_x x/c_\parallel)\}. \tag{24}$$

The spectrum of the Hamiltonian (23) consists of the Landau levels

$$E_{n,\sigma}^\pm = E_n^\pm - h_z\sigma_z$$

split by the component h_z of the magnetic field, parallel to the $\boldsymbol{d}$-vector. Note that

$\mathcal{H}_1^{(1)} = 0$ at both $h_z = 0$ and $h_x = 0$. Considering a general case of $h_x, h_z \neq 0$ and treating $\mathcal{H}_1^{(1)}$ as a perturbation, it can be readily seen that, in the lowest order, diagonal matrix elements $\langle n, \sigma|\mathcal{H}_1^{(1)}|n, \sigma\rangle$ are proportional to $h_z(h_x^2/|\Omega_\perp|)$ and, therefore, represent small corrections due to condition (17). With this accuracy, adopting results of Nersesyan and Vachnadze (1989) for the contribution of the low-energy states to the magnetic field dependence of the ground-state energy (the presence of the Pauli matrix σ_z in the second term in (23) is not essential owing to the $eH \leftrightarrow -eH$ symmetry) and, taking into account both groups of fermions, we obtain

$$\Delta\mathscr{E}(H) = \mathscr{E}(H) - \mathscr{E}(0) = [\zeta(\tfrac{3}{2})/\pi]\nu(H_\perp|\Omega_\perp|^{1/2} - 2\nu(H_\perp)|h_z|[1 + \mathbb{O}(h_x^2/|\Omega_\perp|)]. \tag{25}$$

Here $\nu(H_\perp) = S|eH_\perp|/2\pi c$ is the degree of degeneracy of each Landau level (15) ($S = Na^2$ being the area of the system), and $\zeta(x)$ is Riemann's zeta function.

The first term in (25) describes anomalously strong diamagnetism of the SN state, with the zero-temperature susceptibility

$$\chi_{\mathrm{dia}} \sim -N[3\zeta(\tfrac{3}{2})/4\pi^2](\Delta_0^2/W)(e/m^* c)^{3/2}|H|^{-1/2}\sin^{3/2}\alpha \tag{26}$$

continuously increasing (by the absolute value) with decreasing field†. This increase saturates at temperatures $T \sim |\Omega_\perp|^{1/2}$, when χ_{dia} becomes of the order $-(e^2/m^*c^2)(\Delta_0/T)$.

The second term represents a very important paramagnetic effect caused by non-compensated spins of the $n = 0$ Landau level (Semenoff 1984). Note that it is only contributed by the component of the field, parallel to the $\boldsymbol{d}$-vector.

In the higher-energy region ($\Delta_0 \ll |E| \ll W$), the orbital motion of the particles is quasiclassical and, hence, plays a minor role. Therefore, to obtain the anisotropy energy, it is sufficient to add (14) to the second term of (25):

$$\mathscr{E}_{\mathrm{anis}} = (2N\mu_{\mathrm{B}}^2/\pi W)[(2/\pi)\ln(W/\Delta_0)(\boldsymbol{d}\cdot\boldsymbol{H})^2 - (m/m^*)|\boldsymbol{e}_z\cdot\boldsymbol{H}||\boldsymbol{d}\cdot\boldsymbol{H}|] \tag{27}$$

where m is bare electron mass, $m^* = (1/W)a^2$ is the effective electron mass with the original tight-binding spectrum, and $\boldsymbol{e}_z$ is a unit vector in the real space, perpendicular to the plane of the system. The presence of the invariant in (27), linear in $\boldsymbol{d}$, is in agreement with the general analysis of the anisotropy energy in SN states given by Andreev and Grischuk (1984).

As follows from (27), the equilibrium orientation of the $\boldsymbol{d}$-vector depends on the inclination of the magnetic field and the magnitude of the parameter

$$C = \pi(m/m^*)/[4\ln(W/\Delta_0)]. \tag{28}$$

Minimizing (27) with respect to the angle β between $\boldsymbol{d}$ and $\boldsymbol{H}$ leads to the following results for the equilibrium orientation of the $\boldsymbol{d}$-vector.

(i) If $C > 1$, there exists a finite range of the inclination angle α of the magnetic field given by $\alpha_0 < \alpha < \pi/2$, with $\sin\alpha_0 = 1/C$, when $\beta = 0$ or $\beta = \pi$, i.e. $\boldsymbol{d}$ is oriented along the magnetic field: $\boldsymbol{d} = \pm\boldsymbol{H}/|\boldsymbol{H}|$. In this case, the space of degeneracy for $\boldsymbol{d}$ is Z_2.

† As shown by Nersesyan and Vachnadze (1989), in a quasi-2D (layered) system, the square-root increase in χ_{dia} takes place above some characteristic field $H_0 \sim (m^*c/e)(r_0/b)\Delta_0$ (r_0 is the classical radius of the band electron and b is the interlayer spacing), below which a crossover to a 3D regime, with a finite but anomalously large zero-field diamagnetic susceptibility, takes place. However, since H_0 is extremely small, in fact the condition $H \gg H_0$ is not restrictive.

Certainly, the applicability of equation (27) requires that the angle α_0 should not be too small. Outside this range ($\alpha < \alpha_0$), there are two solutions $\beta = \beta_0$ and $\beta = \pi - \beta_0$, where

$$\cos\beta_0 = C\sin\alpha \tag{29}$$

corresponding to a finite angle between $\boldsymbol{d}$ and $\boldsymbol{H}$. Degenerate states of the $\boldsymbol{d}$-vector form two cones with the rotation axes along $\boldsymbol{H}$ and $-\boldsymbol{H}$. Now the space of degeneracy is $S^1 \times Z_2$.

(ii) If $C < 1$, the equilibrium angle β between $\boldsymbol{d}$ and $\boldsymbol{H}$ is finite at any α and is given by equation (29).

Being restricted by the condition (17), we cannot use equation (29) to describe correctly the limit of $\alpha \to 0$. As in the OAF (Nersesyan and Vachnadze 1989), we expect discontinuities in the angle dependence of the ground state, and hence the anisotropy energy at small α, occurring each time when the spin-up Zeeman sublevels of the n negative-energy Landau levels become empty, while the spin-down sublevels of the n positive-energy levels become filled. Note that, at sufficiently small α, $\mathcal{H}_1^{(1)}$ can no longer be treated as a perturbation, which makes the analysis very complicated. However, bearing in mind the results in section 3, it is qualitatively evident that, in this limit, the angle β between $\boldsymbol{d}$ and $\boldsymbol{H}$ tends to $\pi/2$.

Let us now consider the equilibrium magnetic susceptibility $\chi = \chi_{\text{dia}} + \chi_{\text{para}}$ (where the diamagnetic part is given by equation (26)) of the system at zero temperature. The explicit form of χ_{para} depends on the equilibrium orientation of the $\boldsymbol{d}$-vector. If $\boldsymbol{d}$ is parallel to $\boldsymbol{H}$ ($\beta = 0, \pi$),

$$\chi_{\text{para}} = (2N/\pi W)\,\mu_{\text{B}}(e/m^*c)\sin\alpha \tag{30}$$

and the total susceptibility completely coincides with that in the OAF phase (Nersesyan and Vachnadze 1989). For not too small α, the diamagnetic contribution is dominant:

$$|\chi_{\text{dia}}|/\chi_{\text{para}} \sim (m/m^*)^{1/2}(\Delta_0/\mu_{\text{B}}H)^{1/2} \gg 1.$$

On decreasing α, χ_{para} decreases more slowly than $|\chi_{\text{dia}}|$; so, at $\alpha \sim \mu_{\text{B}}H/\Delta_0$, the paramagnetic response can be dominant. If the equilibrium angle between $\boldsymbol{d}$ and $\boldsymbol{H}$ is either β_0 or $\pi - \beta_0$ (see equation (29)),

$$\chi_{\text{para}} = (8N\mu_{\text{B}}^2/\pi^2 W)\ln(W/\Delta_0)\,(1 + C^2\sin^2\alpha) \tag{31}$$

where parameter C is given by (27). This case is qualitatively similar to the previous one. The only difference is that the dominance of paramagnetism occurs at a relatively larger angle α: $\alpha \sim (\mu_{\text{B}}H/\Delta_0)^{1/3}$.

At finite temperatures the above results are valid, until the temperature is much less than the Zeeman splitting of the $n = 0$ Landau level. However, if $T \gg h$, it can be easily verified that the second term (which is linear in $|\boldsymbol{d}|$) in the anisotropy energy (27) will be changed by $-(N/4\pi W)(m/m^*)(\mu_{\text{B}}H/T)\mu_{\text{B}}^2\,(\boldsymbol{d}\cdot\boldsymbol{H})^2$, which represents a small correction to the first term in (27). We thus conclude that, on increasing the temperature and reaching the range $T > \mu_{\text{B}}H$, the angle between $\boldsymbol{d}$ and $\boldsymbol{H}$ continuously increases and eventually becomes equal to $\pi/2$, irrespective of the field's orientation relative to the plane of the system.

5. Role of spin–orbit interaction

In this section we consider the effect of weak spin–orbit interaction on the orientation of the d-vector. We use the following, purely two-dimensional form of single-particle spin–orbit interaction-preserving translational symmetry of the square lattice:

$$H_{so} = \sum_k c^+_{k\alpha} \Lambda(k) \cdot \sigma_{\alpha\beta} c_{k\beta} \tag{32}$$

where, because of the requirement of hermiticy, $\Lambda(k)$ is a real vector function of k.

As already mentioned, we assume that the two-cosine tight-binding spectrum $\varepsilon(k)$ is orbitally non-degenerate. This corresponds to zero average value of electron's orbital moment at each lattice site, in which case diagonal elements of the spin–orbit operator in the Wannier representation vanish because of time reversal invariance. This means that spin–orbit effects can only be revealed as a result of the particle intersite hopping on the square lattice. In the simplest case such a situation corresponds to an s-wave band. It also appears to be the case in high-T_c materials with Cu–O layers. Owing to the crystal field and Jahn–Teller splitting of the originally degenerate 3d electron states on the Cu atoms, the highest-energy state that forms a two-dimensional half-filled band is well separated from other levels and has $3d_{x^2-y^2}$ symmetry with quenched orbital moment ($\langle l \rangle = 0$).

We thus find that the requirement of time reversal invariance implies that $\Lambda(-k) = -\Lambda(k) \cdot \Lambda(k)$ must also transform as a pseudovector under the point group D_4 of the square lattice. For coinciding reference frames in real and spin spaces, the simplest form of $\Lambda(k)$ corresponding to the nearest-neighbour approximation for the electron hopping, which has already been used for the original tight-binding spectrum, is

$$\Lambda(k) = (\Lambda_0/\sqrt{2})[e_x \sin(k_y a) - e_y \sin(k_x a)] \tag{33}$$

where Λ_0 is a real parameter: $\Lambda_0 \ll \Delta_0$.

In the presence of the spin–orbit interaction, the spectrum is given by

$$E^2(k) = \varepsilon^2(k) + \Delta^2(k) + \Lambda^2(k) \pm 2\{[\varepsilon^2(k) + \Delta^2(k)]|\Lambda|^2(k) \\ - \Delta^2(k)[d \cdot \Lambda(k)]^2\}^{1/2}. \tag{34}$$

If d is perpendicular to the plane of the system ($d = e_z$), a splitting of the spectrum (6) takes place, and a finite Fermi surface appears, with the density of states at zero energy proportional to Λ_0. If d lies in the plane of the system, the spin–orbit interaction keeps the Fermi surface zero dimensional, only splitting the zeros of the original spectrum (6). In full analogy with the case for a longitudinal magnetic field (see section 3), one thus concludes that the spin-orbit interaction lies in the plane of the d-vector. Indeed, treating (32) as a perturbation with respect to the mean-field Hamiltonian (4), in the second order in Λ_0 we obtain the following spin–orbit part of the anisotropy energy:

$$\mathscr{E}_{so} = \frac{\Lambda_0^2}{8}(d \cdot \sigma_z)^2 \sum_k \frac{\Delta^2(k)[\sin^2(k_x a) + \sin^2(k_y a)]}{E_0^3(k)} \\ = N\frac{\Lambda_0^2}{W}(d \cdot e_z)^2\left[1 + \mathcal{O}\left(\frac{\Delta^2}{W^2}\right)\right]. \tag{35}$$

Possible anisotropy of the exchange interaction between electrons on neighbouring sites also affects the orientation of the $\boldsymbol{d}$-vector. The corresponding contribution to the ground-state energy of the SN state is proportional to

$$\Delta\mathscr{E}_{\text{exch}} \sim N\gamma(\Delta_0/W)^2(\boldsymbol{d}\cdot\boldsymbol{e}_z)^2 \tag{36}$$

where γ is the exchange anisotropy constant. Depending on the sign of γ and relationship between the parameters, equations (35) and (36) result either in an 'easy-axis' anisotropy, or in an 'easy-plane' anisotropy. The space of degeneracy of the $\boldsymbol{d}$-vector is Z_2 or S^1, respectively.

Let us consider now the combined effect of the spin–orbit interaction and the magnetic field. Clearly, in the presence of the magnetic field parallel to the plane, the spin–orbit interaction results in the appearance of in-plane easy-axis anisotropy, which reduces the SO(2) degeneracy of the SN state down to $Z_2 : \boldsymbol{d} = \pm(\boldsymbol{H}\times\boldsymbol{e}_z)/|\boldsymbol{H}|$.

For a finite inclination of the field, the most interesting situation arises when the condition

$$|\Omega_\perp|^{1/2} \gg \Lambda_0 \gg |h| \tag{37}$$

is satisfied. Now we shall show that the splitting of the $n = 0$ Landau level, caused by the spin–orbit interaction, results in a contribution to $\mathscr{E}_{\text{anis}}$, which is linear in both H and Λ_0 and leads to a two-axis in-plane anisotropy.

The contribution of the 'high'-energy region ($\Delta_0 \ll |E| \ll W$) is still given by equation (35). Neglecting the Zeeman splitting of the single-particle levels and considering for simplicity the case of a perpendicular field, the low-energy ($|E| \ll \Delta_0$) one-dimensional Hamiltonians for the two groups of fermions will take the form

$$\mathscr{H}_1 = (c_\parallel\hat{p}_x - \Lambda_0\sigma_y)\tau_x - (eH_\perp c_\perp\tilde{x}/c)(\boldsymbol{d}^*\cdot\boldsymbol{\sigma})\tau_y \tag{38}$$

$$\mathscr{H}_2 = (c_\parallel\hat{p}_y + \Lambda_0\sigma_x)\tau_x + (eH_\perp c_\perp\tilde{y}/c)(\boldsymbol{d}^*\cdot\boldsymbol{\sigma})\tau_y \tag{39}$$

where $\boldsymbol{d}^*$ is obtained from $\boldsymbol{d}$ by a $\pi/4$ rotation†:

$$\boldsymbol{d}^* = ((d_x + d_y)/\sqrt{2}, (-d_x + d_y)/\sqrt{2}, d_z).$$

The symmetry properties

$$x \leftrightarrow y \qquad y \leftrightarrow -x \qquad d_x \to -d_y \qquad d_y \to d_x$$
$$\psi_1 \to \tau_x\psi_2 \qquad \psi_2 \to \tau_y\psi_1 \qquad \mathscr{H}_1 \leftrightarrow \mathscr{H}_2 \tag{40}$$

allow one to consider $\mathscr{H}_1$ only.

Let us characterize the orientation of vector $\boldsymbol{d}^*$ in real space by the spherical angles ϑ and $\varphi : d_x^* = \sin\vartheta\cos\varphi,\ d_y^* = \sin\vartheta\sin\varphi,\ d_z^* = \cos\vartheta$. Making then a rotation of the spin space,

$$\psi_1 \to U^{-1}\psi_1 \qquad U(\vartheta,\varphi) = \exp[(\mathrm{i}/2)\vartheta(\sigma_x\sin\varphi - \sigma_y\cos\varphi)]$$

to direct the transformed z axis along $\boldsymbol{d}^*$, after a proper rotation around $\hat{z}$ we obtain

$$\mathscr{H}_1 \to [c_\parallel\hat{p}_x + \Lambda_0(1 - d_y^{*2})^{1/2}\sigma_x - \Lambda_0 d_y^*\sigma_z]\tau_x - (eH_\perp c_\perp x/c)\sigma_z\tau_y.$$

Using now a transformation $\psi_1 \to V\psi_1$ with $V = (1/\sqrt{2})(1 + \mathrm{i}\sigma_y\tau_x)$ results in

$$\mathscr{H}_1 = c_\parallel\hat{p}_x\tau_x - (eH_\perp c_\perp x/c)\sigma_z\tau_y + \Lambda_0(1 - d_y^{*2})^{1/2}\sigma_z + \Lambda_0 d_y^*\sigma_x. \tag{41}$$

The structure of the Hamiltonian (41) is identical with (19); therefore, according to

† The transformation $\boldsymbol{d} \to \boldsymbol{d}^*$ should have been assumed in equation (7). However, it is unimportant in the absence of the spin–orbit interaction when there is no coupling between the real and spin spaces.

(25), its contribution to the anisotropy energy equals $\nu(H_\perp)|\Lambda_0|(1-d_y^{*2})^{1/2}$. Because of the symmetry property (40), the contribution of $\mathcal{H}_2$ differs from that in (40) only in that d_y^{*2} should be changed to d_x^{*2}. Taking into account (35), for the anisotropy energy we obtain

$$\mathscr{E}_{\text{anis}} = (N/W)\{\Lambda_0^2 d_z^{*2} - (1/\pi)(m/m^*)|\mu_B H||\Lambda_0|\sin\alpha\,[(1-d_x^{*2})^{1/2} + (1-d_y^{*2})^{1/2}]\}. \tag{42}$$

From (42), one concludes that, under the condition (37), the minimum value of $\mathscr{E}_{\text{anis}}$ is achieved at $d_z^* = 0$, $|d_x^*| = |d_y^*| = 1/\sqrt{2}$. This corresponds to two-axis in-plane anisotropy which orients the $\boldsymbol{d}$-vector along either of the crystallographic axes of the square lattice. The space of degeneracy is Z_4.

6. Conclusion

In this paper, we have discussed the low-temperature magnetic properties of one of the possible states of 2D interacting Fermi system—the SN state. Using a simple mean-field description, we have shown that the most interesting effects arise in a magnetic field affecting the orbital motion of gapless fermionic excitations, whose existence is dictated by violation of the invariance with respect to $\pi/2$ rotations from the point group D_4 in the SN phase. Among these effects are anomalous diamagnetism, similar to that in the OAF state, and the important contribution to the anisotropy energy which determines the space of degeneracy for the spin vector $\boldsymbol{d}$ of the order parameter.

As we have seen, in different conditions, this space may be discrete (Z_2, Z_4) or continuous (S^2, S^1 and $S^1 \times Z_2$). In the former case, an Ising-like symmetry will result in the existence of long-range order below some characteristic temperature even in a purely two-dimensional system. For a continuously degenerate SN state, a mean-field approach can only be applicable to estimate the crossover temperature below which the amplitude fluctuations of the local order parameter become small (Schulz 1989a). To describe the orientational dynamics of the $\boldsymbol{d}$-vector at low temperatures ($T \ll \Delta_0$), one would have first to integrate off the fermionic degrees of freedom to obtain, in the continuum limit, the free-energy functional. It would be very interesting to perform this calculation taking into account the 'relativistic' Landau quantization of the low-energy states in the magnetic field.

Various types of degeneracy space of the SN order parameter allow different topologically stable textures of the vector field $\boldsymbol{d}(\boldsymbol{x})$, e.g. domain walls in the case of Z_2 symmetry, or XY-model-like vortices in the case of SO(2) symmetry. The electronic structure of topological defects in the 2D commensurate SN phase is now under study.

In this paper, we have described the low-temperature magnetic properties of a commensurate SN phase, with a homogeneous orientation of the $\boldsymbol{d}$-vector, for weakly interacting electrons with a simple, exactly half-filled energy band. There remains the question of stability of such a phase with respect to possible changes in the original model that violate the perfect nesting property of the 2D Fermi surface. This occurs, for example, when one takes into account next-nearest-neighbour hopping of the particles, or when finite deviations from the band's half-filling are considered. In the latter case, by analogy with 2D antiferromagnets, described by the weak-coupling Hubbard model (Schulz 1989b) one might imagine the development of an incommensurate SN phase via the formation of a domain wall structure of the $\boldsymbol{d}$-vector under doping, with the added holes localized at the walls. However, the existence of the low-energy excitations in the SN phase can essentially change this picture, since the stability of such domain walls is not clear. We hope to investigate these questions in the future.

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Appendix

The extended 'U–V–J' Hubbard model is given by the Hamiltonian

$$\hat{H} = -t \sum_{\langle n,m \rangle} c^{+}_{n\alpha} c_{m\alpha} + U \sum_{n} n_{n\uparrow} n_{n\downarrow} + V \sum_{\langle n,m \rangle} n_n n_m + J \sum_{\langle n,m \rangle} s_n \cdot s_m \tag{A1}$$

where the symbol $\langle n, m \rangle$ denotes nearest-neighbour sites, $n_n = c^{+}_{n\alpha} c_{n\alpha}$ and $s_n = \frac{1}{2} c^{+}_{n\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{n\beta}$. We shall be interested in the mean-field instabilities in this model at half-filling, assuming that U, $V > 0$ and thus excluding the superconducting instability from the consideration. The local operators whose expectation values determine the order parameters for the CDW, SDW, OAF and SN states, are given by

$$\begin{aligned} O_{\mathrm{CDW}}(n) &= (-1)^{n_x + n_y} c^{+}_{n\alpha} c_{n\alpha} \\ O_{\mathrm{SDW}}(n) &= (-1)^{n_x + n_y} c^{+}_{n\alpha} \boldsymbol{d} \cdot \boldsymbol{\sigma}_{\alpha\beta} c_{n+g,\beta} \\ O_{\mathrm{OAF}}(n, n+g) &= \mathrm{i}(-1)^{n_x + n_y} \lambda(g) c^{+}_{n\alpha} c_{n+g,\alpha} \\ O_{\mathrm{SN}}(n, n+g) &= \mathrm{i}(-1)^{n_x + n_y} \lambda(g) c^{+}_{n\alpha} \boldsymbol{d} \cdot \boldsymbol{\sigma}_{\alpha\beta} c_{n+g,\beta} \end{aligned} \tag{A2}$$

where $g = \pm a_x, \pm a_y, \lambda(\pm a_x) = -\lambda(\pm a_y) = 1, |d|^2 = 1$.

Using the standard mean-field procedure for each order parameter independently, one obtains self-consistency equations for the corresponding amplitudes Δ_{i} (i ≡ CDW, SDW, OAF, SN):

$$\Delta_{\mathrm{i}} \left[1 - \frac{\Gamma_{\mathrm{i}}}{2N} \sum_{k} \frac{\phi_{\mathrm{i}}(k)}{E_{\mathrm{i}}(k)} \tanh\left(\frac{E_{\mathrm{i}}(k)}{2T} \right) \right] = 0. \tag{A3}$$

Here Γ_{i} are linear combinations of the interaction constants

$$\Gamma_{\mathrm{CDW}} = 8V - U \qquad \Gamma_{\mathrm{SDW}} = U + 4J$$
$$\Gamma_{\mathrm{SN}} = 4V - 2J \qquad \Gamma_{\mathrm{OAF}} = 4V + 6J \tag{A4}$$

the function $\phi(k) = 1$ for CDW and SDW, and $\phi(k) = \frac{1}{4} (\cos k_x - \cos k_y)^2$ for OAF and SN; $E_i(k) = [\varepsilon^2(k) + \Delta_i^2(k)]^{1/2}$, where $\Delta_i(k) = \Delta_i$ for CDW and SDW, and $\Delta_i(k) = \Delta_i(\cos k_x - \cos k_y)$ for OAF and SN.

At $\Gamma_i > 0$, equations (A3) define the corresponding mean-field transition temperature which can be roughly estimated within $\log^2 T$ accuracy, i.e. by taking into account only those states close to the Van Hove saddle points of the spectrum $\varepsilon(k)$, $k_A = (\pi/a, 0)$ and $k_B = (0, \pi/a)$: $T_{c;i} \simeq 8t \exp[-(8\pi^2 t/\Gamma_i)^{1/2}]$. The dominating instability is determined by the largest positive Γ_i.

In the absence of the exchange interaction, i.e. $J = 0$ (the 'U–V' model), one has $\Gamma_{CDW} > \Gamma_{OAF} = \Gamma_{SN} > \Gamma_{SDW}$ at $U < 4V$, and $\Gamma_{SDW} > \Gamma_{OAF} = \Gamma_{SN} > \Gamma_{CDW}$ at $U > 4V$, thus observing the CDW-to-SDW-phase transition on increasing U from the region $U < 4V$ to the region $U > 4V$ (see e.g., Zhang and Callaway 1989). On the critical line $U = 4V$ all Γ_i equal $4V$, indicating degeneracy between the transition temperatures for the density waves and the current states. Certainly, the mean-field approximation is not adequate for singling out the dominant instability at $U = 4V$, since the interference between different competing instabilities is totally ignored in this scheme. However, as follows from (A4), the above degeneracy is removed by a weak exchange interaction, and for any sign of J the dominant instability turns out to be either of the OAF type ($U = 4V$, $J > 0$) or of the SN type ($U = 4V$, $J < 0$). We believe that the validity of this conclusion, which should also hold in some vicinity of the critical line $U = 4V$, goes beyond the mean-field approximation.

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